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Generating functional and large N limit of nonlocal 2D generalized Yang–Mills theories (nlgYM$_2$'s)

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Abstract. Using the path integral method, we calculate the partition function and the generating functional (of the field strengths) on nonlocal generalized $2D$ Yang–Mills theories (nlgYM$_2$'s), which are nonlocal in the auxiliary field. This has been considered before by Saaidi and Khorrami. Our calculations are done for general surfaces. We find a general expression for the free energy of $W(\phi) = \phi^{2k}$ in nlgYM$_2$ theories at the strong coupling phase (SCP) regime ($A > A_c$) for large groups. In the specific ϕ^4 model, we show that the theory has a third order phase transition.

1 Introduction

This paper will be devoted to a renewed study of two-dimensional Yang–Mills theory without matter, a system which can easily be solved. Yet we will see that there is still much to say about this system. Pure two-dimensional Yang–Mills theories (YM$_2$'s) have certain properties, such as invariance under an area preserving diffeomorphism and lack of any propagating degrees of freedom [6]. There are, however, ways to generalize these theories without losing those properties. One way, leading to what are called generalized Yang–Mills theories (gYM$_2$'s) [2], is to write

$$\mathrm{iTr}(B\epsilon^{\mu\nu}F_{\mu\nu}) + f(B). \tag{1}$$

Here $F_{\mu\nu}$ is the Yang–Mills field strength and B is a scalar field in the adjoint representation of the gauge group. Standard dimensional analysis applied to (1) shows that $F_{\mu\nu}$ has dimension 2 and B dimension 0, so power counting allows an arbitrary class function $f(B)$. In this model, produced by Witten [2], and one obtains the partition function by considering its action as a perturbation of the topological theory at zero area. In [3–5] the Green function, partition function and expectation values of Wilson loops were calculated. One can, however, use standard path integration and calculate the observables of the theory [6,12]. To study the behavior of these theories for large groups is also interesting. This was done in [8–11] for ordinary YM$_2$ theories and in [12,13] for gYM$_2$ theories. It was shown that YM$_2$'s and some classes of gYM$_2$'s have a third order phase transition in a certain area. There is another way to generalize YM$_2$ and gYM$_2$, and that is to use a nonlocal action for the auxiliary field, leading to the so-called nonlocal YM$_2$ (nlYM$_2$'s) and nonlocal gYM$_2$ (nlgYM$_2$'s) theories, respectively [14]. The authors of [14] studied nlYM$_2$ and investigated the order of the transition for that case. We want to study the wave function, partition function, generating functional of nlgYM$_2$ and also their properties for a large gauge group in the state in which $W(\phi) = \phi^4$. The scheme of the present paper is the following.

In Sect. 2, the wave function and partition function of nlgYM$_2$ on general surfaces are computed. In Sect. 3, the generating functional of nlgYM$_2$ on a disk and on general surfaces are calculated. In Sect. 4, the properties of nlgYM$_2$ large groups, for the case in which $f(B) = \mathrm{Tr}(B^{2k})$, are studied. Finally in Sect. 5, we test our theory for the ϕ^4 model ($f(B) = \mathrm{Tr}(B^4)$). It is shown that the large group properties of nlgYM$_2$ are the same as was found for ordinary gYM$_2$.

2 The wave function of nlgYM$_2$

The nlgYM$_2$ is defined by [14]

$$\mathrm{e}^S := \int DB \exp\left\{\mathrm{i}\int \mathrm{Tr}(BF)\mathrm{d}\mu + \omega\left[\int f(B)\mathrm{d}\mu\right]\right\}, \tag{2}$$

where $\mathrm{d}\mu$ is the invariant measure of the surface

$$\mathrm{d}\mu := \frac{1}{2}\epsilon_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu. \tag{3}$$

F is the field strength corresponding to the gauge field and B is a pseudo-scalar field in the adjoint representation of the group. Along the line of [12,14], we begin by calculating the wave function on a disk. We obtain

$$\psi_D(U) = \int DF \mathrm{e}^S \delta\left(P \exp \oint_{\partial D} A, U\right). \tag{4}$$

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Here U is the class of the Wilson loop corresponding to the boundary. The delta function is also a class delta function; its support agrees with the boundary conditions. This delta function can be expanded in terms of the characters of irreducible unitary representations of the group; i.e.

$$\delta\left(P\exp\oint_{\partial D} A, U\right) = \sum_R \chi_R(U^{-1})\chi_R\left(P\exp\oint_{\partial D} A\right). \tag{5}$$

We introduce fermionic variables η and $\bar{\eta}$ in the representation R to write the Wilson loop as [6,12]

$$\chi_R\left(P\exp\oint_{\partial D} A\right) = \int D\eta D\bar{\eta}\exp\left\{\int_0^1 \mathrm{d}t\bar{\eta}(t)\dot{\eta}(t) + \oint_{\partial D}\bar{\eta}A\eta\right\}\eta^\alpha(0)\bar{\eta}_\alpha(1). \tag{6}$$

Inserting (6) in (5) and then (4), using the Schwinger–Fock gauge, and integrating over F, B, and the fermionic variables, respectively, one obtains

$$\psi_D(U) = \sum_R \chi_R(U^{-1})d_R\exp\{\omega[AC_f(R)]\}; \tag{7}$$

here d_R is the dimension of the representation R and

$$C_f(R)1_R =: f(-\mathrm{i}T_R). \tag{8}$$

$f(-\mathrm{i}T_R)$ means that one has put $-\mathrm{i}T^a$ in the representation R instead of B^a in the function f. The action of the original B–F theory (2) is not extensive; i.e.

$$S_{A_1+A_2}(B,F) \neq S_{A_1}(B,F) + S_{A_2}(B,F). \tag{9}$$

Therefore, one cannot simply glue the disk wave function to obtain the wave function corresponding to a larger disk. To obtain the wave function for an arbitrary surface, however, one can begin with a disk of the same area and impose boundary conditions on certain parts of the boundary of the disk. These conditions correspond to the identifications needed for constructing the desired surface from a disk. The only things to be calculated are integrations over the group of characters of the same representation [5]. This is easily done and one arrives at

$$\psi_{\sum_{g,q}}(U_1,\ldots,U_n) = \sum_R h_R^q d_R^{2-2g-q-n} \times\chi_R(U_n^{-1})\ldots\chi_R(U_n^{-1})\exp\left\{\omega\left[-C_f(R)A_{\sum_{g,q}}\right]\right\}, \tag{10}$$

where $\sum_{g,q}$ is a surface containing g handles, n boundaries and q projective planes. h_R is defined as

$$h_R := \int \mathrm{d}U\chi_R(U^2); \tag{11}$$

$h_R = 0$ unless the representation R is self-conjugate. In this case, this representation has an invariant bilinear form. Then, $h_R = 1$ if this form is symmetric and $h_R = -1$ if it is antisymmetric [15].

The partition function of the theory on a sphere is obtained if we put the U_i's equal to unity and g and q equal to zero. We obtain

$$Z_{s^2} = \sum_R d_R^2\exp\{\omega[-AC_f(R)]\}. \tag{12}$$

3 The generating functional $Z[J]$ of nlgYM$_2$

To calculate the Green functions of the F^a's, we again begin with the disk and calculate the wave function of nlgYM$_2$ on the disk, with a source term coupled to F; i.e.

$$\psi_D[J] = \int DF\mathrm{e}^{\{S+\int\mathrm{Tr}(FJ)\mathrm{d}\mu\}}\delta\left(P\exp\oint_{\partial D} A, U\right). \tag{13}$$

Following the same steps as in the previous section, we arrive at

$$\psi_D[J] = \sum_R \chi_R(U^{-1}) \times \mathrm{Tr}_R\left\{P\exp\left(\omega\left[\int f(\mathrm{i}J^a(x)+\mathrm{i}T^a)\mathrm{d}\mu\right]\right)\right\}. \tag{14}$$

In the above equation P stands for ordering according to the angle variable on the disk. To obtain the generating functional $Z[J]$ of nlgYM$_2$ for an arbitrary surface, $\sum_{g,q}$, we can use the same procedure as was used in obtaining (11) and the result is

$$Z_{\sum_{g,q}}[J] = \sum_R h_R^q d_R^{2-2g-q-1}\exp\{\omega[AC_f(R)]\} \times \mathrm{Tr}_R\left\{P\exp\left(\omega\left[\int f(\mathrm{i}J^a+\mathrm{i}T^a)\mathrm{d}\mu\right]\right)\right\}. \tag{15}$$

As an example, consider YM$_2$, in which $\omega\left[\int f(B)\mathrm{d}\mu\right] = -1/2\epsilon\int\mathrm{Tr}(B^2)\mathrm{d}\mu$. In this case (15) reduces to

$$Z_{\sum_{g,q}}[J] = Z_1[J]\sum_R h_R^q d_R^{2-2g-q-1} \times\exp\left\{-\frac{\epsilon}{2}C_2(R)A_{\sum_{g,q}}\right\} \times\mathrm{Tr}_R\left\{P\exp\left(\epsilon\int\mathrm{d}t\int\mathrm{d}s\sqrt{g}J(t,s)\right)\right\} \tag{16}$$

where

$$Z_1[J] = \exp\left(-\frac{\epsilon}{2}\int J^aJ_a\mathrm{d}\mu\right).$$

This is in agreement with the result obtained in [12]. Functional differentiating of (15) with respect to $J(x)$ gives us the n-point functions of the F's in the Schwinger–Fock gauge.

4 Large N limit of nlgYM$_2$

Starting from (12), consider the case that the gauge group is $U(N)$. The representations of this group are labeled by N integers n_i satisfying

$$n_i \geq n_j, \qquad i \leq j. \tag{17}$$

The dimension of this representation is

$$d_R = \prod_{1\leq i\leq j\leq N}\left(1+\frac{n_i-n_j}{j-i}\right), \tag{18}$$

and the kth Casimir operator is

$$C_k(R) = \sum_{i=1}^{N}[(n_i+N-i)^k-(N-i)^k]. \tag{19}$$

Taking for $C_f(R)$ a linear function of the Casimir operators (19), redefining the function ω and introducing another function by

$$-N^2V\left[A\sum_{k=1}^{N}a_k\hat{C}_k(R)\right] := \omega[-AC_f(R)], \tag{20}$$

where

$$\hat{C}_k(R) = \frac{1}{N^{k+1}}\sum_{i=1}^{N}(n_i+N-i)^k, \tag{21}$$

then, following [10], we can use the definitions

$$x := \frac{i}{N}, \tag{22}$$

and

$$\phi(x) = \frac{i-n_i-N}{N}. \tag{23}$$

So apart from an unimportant constant, the partition function takes the form

$$Z[\phi(x)] = \int D\phi(x)\mathrm{e}^{\{-N^2S(\phi)\}}, \tag{24}$$

where

$$S(\phi) = V\left(A\int_0^1 W[\phi(x)]\mathrm{d}x\right) + \int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\log|\phi(x)-\phi(y)|, \tag{25}$$

and

$$W(\phi) := \sum_{k=1}(-1)^k a_k\phi^k. \tag{26}$$

In the large N limit, only the configuration of ϕ contributes to the partition function that minimizes S. To find it, we put the variation of S with respect to ϕ equal to zero. Then

$$\frac{\hat{A}}{2}W'(\phi) = P\int_0^1\frac{\mathrm{d}t}{\phi(x)-\phi(x)}, \tag{27}$$

where

$$\hat{A} := AV'\left[A\int_0^1\mathrm{d}xW(\phi(x))\right]. \tag{28}$$

One defines a density function for ϕ BY

$$u(\phi) := \left.\frac{\mathrm{d}x(\phi)}{\mathrm{d}\phi}\right|_{\phi=z}, \tag{29}$$

which should be positive and normalized to

$$\int_{-a}^{a}u(z)\mathrm{d}z = 1. \tag{30}$$

Then (27) becomes

$$\frac{\hat{A}}{2}W'(z) = P\int_{-a}^{a}\frac{u(t)\mathrm{d}t}{z-t}. \tag{31}$$

To solve (31), we defined the function $H(z)$ on the complex z-plane [10]

$$H(z) := \int_{-a}^{a}\frac{u(t)\mathrm{d}t}{z-t}. \tag{32}$$

This function is analytic on the complex plane, except for a cut at $[-a,a]$. With proceed by the same procedure as was followed in [12], and we arrive at

$$H(z) = \frac{\hat{A}}{2}W'(z) - \sqrt{z^2-a^2} \times \sum_{m,n=0}^{\infty}M_n\frac{a^{2n}z^m}{(2n+m+1)!}g^{(2n+m+1)}(0), \tag{33}$$

where

$$g(z) = \frac{\hat{A}}{2}W'(z), \tag{34}$$

and

$$M_n = \frac{(2n-1)!!}{2^n n!}, \qquad M_0 = 1. \tag{35}$$

$g^{(k)}$ is the kth derivative of g with respect to z. From (32), it is seen that

$$\mathrm{Im}H(z+\mathrm{i}\epsilon) = -\pi u(z), \qquad x\in[-a,a], \tag{36}$$

which gives

$$u(z) = \frac{\sqrt{a^2-z^2}}{\pi}\sum_{n,m=0}^{\infty}\frac{M_n a^{2n}z^m g^{(2n+m+1)}(0)}{(2n+m+1)!}. \tag{37}$$

To obtain a, one can use (30) and (37), which yields

$$\sum_{n=0}^{\infty}\frac{M_n a^{2n}g^{(2n-1)}(0)}{(2n-1)!} = 1. \tag{38}$$

Defining a free energy function by

$$F := -\frac{1}{N^2}S|_{\phi_{\mathrm{cla.}}}, \tag{39}$$

it is seen that

$$F'(A) = V'(A\kappa)\kappa, \tag{40}$$

where

$$\kappa = \int_0^1 W[\phi(x)]\mathrm{d}x = \int_{-a}^{a}u(z)W(z)\mathrm{d}z. \tag{41}$$

By making use of (37) and the explicit expression for $W(z)$ as a function of z, we can calculate κ and therefore at last we can compute $F'_{\mathrm{w}}(A)$ (40) for this model. Note that the above solution is valid in the weak ($A\leq A_{\mathrm{c}}$) regime, where A_{c} is the critical area. If $A>A_{\mathrm{c}}$, then the constraint $u\leq 1$ is violated.

5 The $W(z) = z^{2k}$ model for nlgYM$_2$

5.1 WCP regime ($A \leq A_c$)

In order to study the behavior of any model in the SCP regime ($A > A_c$), we need to know the explicit form of the density function in the weak regime, $u_w(z)$. So by rewriting (37), (38) and (40) for the z^{2k} model, one arrives at

$$u_w(z) = \frac{k\hat{A}}{\pi}\sqrt{a^2 - z^2}\sum_{n=0}^{k-1} M_n a^{2n} z^{2k-2n-2}, \quad (42)$$

$$\frac{k\hat{A}a^{2k}}{2^k}Q(k) = 1, \quad (43)$$

$$F'_w(A) = \frac{kV'\hat{A}a^{4k}}{2^k}E(k), \quad (44)$$

where

$$Q(k) = \sum_{n=0}^{k-1} \frac{(2k-2n-3)!!(2n-1)!!}{(k-n-1)!(n+1)!},$$
$$E(k) = \sum_{n=0}^{k-1} \frac{(2k-2n-3)!!(2k+2n-1)!!}{(k-n-1)!(k+n+1)!}. \quad (45)$$

This is, of course, in complete accordance with [13]. But one must now obtain the quantities in terms of A, not $\hat{A}$. It is seen that

$$F'_w(A) = \frac{E(k)}{kAQ^2(k)} = \frac{1}{2kA}. \quad (46)$$

The function V has disappeared from $F'_w(A)$, as can be seen by the rescaling $\hat{\phi} := A^{1/(2k)}\phi$.

This completes our discussion of the weak-region nlgYM$_2$. As A increases, a situation is encountered where u_w exceeds 1. This density function is, however, not acceptable, as it violates the condition (17).

5.2 SCP regime ($A > A_c$)

One of the interesting points of the $Z^{2k}(k > 1)$ model is the fact that the density function in the weak region, (42), has only one minimum at $z = 0$, and two maxima which are symmetric with respect to the origin [13]. So, to find the density function in the strong region will be relevant for the three cut Cauchy problem. Hence following [12], we use the following ansatz for u_s

$$u_s(z) = \begin{cases} \hat{u}_s(z), & z \in L := [-a,-b] \cup [-c,c] \cup [b,a], \\ 1, & z \in L' := [-b,-c] \cup [c,b]. \end{cases} \quad (47)$$

Using methods exactly the same as those used in [12], one must solve

$$\frac{\hat{A}}{2}W'(z) = P\int_{-a}^{a} \frac{u_s(t)\mathrm{d}t}{z-t}, \qquad z \in L, \quad (48)$$

and

$$\int_c^b \left\{\frac{\hat{A}}{2}W'(z) - P\int_{-a}^{a}\frac{u_s(t)\mathrm{d}t}{z-t}\right\}\mathrm{d}z = 0. \quad (49)$$

To do so, one defines a function H_s by

$$H_s(z) = \int_{-a}^{a} \frac{u_s(t)\mathrm{d}t}{z-t}, \quad (50)$$

which is found to be

$$H_s(z) = k\hat{A}z^{2k-1} + 2T(z)\left[k\frac{\hat{A}}{2}\sum'_{[n_i]=0}\tau(n_1,n_2,n_3)z^{2n_4} - \int_c^b \frac{t\mathrm{d}t}{(z^2-t^2)T(t)}\right], \quad (51)$$

where the prime on $\sum$ indicates the following condition:

$$\sum_{i=1}^{4} n_i = k-2, \quad (52)$$

and

$$T(z) = \sqrt{(a^2-z^2)(b^2-z^2)(c^2-z^2)}, \quad (53)$$
$$\tau(n_1,n_2,n_3) = M_{n_1}M_{n_2}M_{n_3}a^{2n_1}b^{2n_2}c^{2n_3}. \quad (54)$$

Using the fact that $H_s(z)/T(z)$ should behave as $1/z^4$ for large z, one obtains

$$k\hat{A}\sum'_{[n_i]=0}\tau(n_1,n_2,n_3) = 2\int_c^b \frac{t\mathrm{d}t}{T(t)}, \quad (55)$$

$$k\hat{A}\sum'_{[n_i]=0}\tau(n_1,n_2,n_3) = 1 + 2\int_c^b \frac{t^3\mathrm{d}t}{T(t)}. \quad (56)$$

Here the prime over the summations in (55) and (56) indicates the following conditions, respectively:

$$\sum_{i=1}^{3} n_i = k-1, \quad (57)$$
$$\sum_{i=1}^{3} n_i = k. \quad (58)$$

In order to obtain the parameters a, b and c in spite of (55) and (56) we need another equation (50) which is found by expressing the action in terms of $u_s(z)$ and minimizing this along with (30) as a constraint [11,12]. By expanding (50) and (51) at large z and comparing them, one can easily arrives at

$$F'_s(A) = V'_s(A\kappa_s)\left\{k\hat{A}\sum'_{[n_i]=0}\tau(n_1,n_2,n_3)\tau_1(n_4,n_5,n_6) + 2\sum'_{[n_i]=0}\tau_1(n_1,n_2,n_3)\int_c^b \frac{t^{2n_4+1}\mathrm{d}t}{T(t)}\right\}, \quad (59)$$

where the prime over the first and second summation indicates the following constraints, respectively:

$$\sum_{i=1}^{6} n_i = 2k, \tag{60}$$

$$\sum_{i=1}^{4} n_i = k+1, \tag{61}$$

and

$$\tau_1(n_1,n_2,n_3) = \frac{a^{2n_1}b^{2n_2}c^{2n_3}}{2^{n_1+n_2+n_3}}\prod_{i=1}^{3}\frac{(2n_i-3)!!}{n_i!}, \tag{62}$$

where we define $(-3)!! = -1$.

Equation (59) is an explicit relation for $F'_s(A)$, which represents the SCP regime of our theory. It is seen that the structure of $F'_s(A)$ is very complicated; therefore, as an example, we study the order of the transition for the z^4 model ($k = 2$).

6 The z^4 model of nlgYM$_2$

6.1 WCP regime ($A \leq A_c$)

In the previous section we studied the nlgYM$_2$ for the z^{2k} model. In this section we can check the result for the z^4 model. By rewriting (42)–(45), we have

$$u_w(z) = \frac{\hat{A}}{\pi}\sqrt{a^2 - z^2}(a^2 + 2z^2), \tag{63}$$

$$\kappa_w = \frac{3a^4}{16}, \tag{64}$$

$$\hat{A} = \frac{4}{3a^4}, \tag{65}$$

and

$$F'_w(A) = \frac{1}{4A}. \tag{66}$$

It is seen that the density function in the WCP regime, $u_w(z)$, has a minimum at $z = 0$, and two maxima at $z_{1,2} = \pm a/2^{1/2}$. Equations (63)–(66) are valid in the regime in which $a \leq a_c = 8/(3(2^{1/2})\pi)$ or $A \leq A_c$. The value of A_c is obtained from

$$u_w(z_{1,2}) = 1, \tag{67}$$

which gives

$$A_c V'_c\left(\frac{32A_c}{27\pi^4}\right) = \frac{27\pi^4}{256}. \tag{68}$$

In spite of some constant, these almost are the same results as have been calculated for the local gYM$_2$ theory [12].

6.2 SCP regime ($A > A_c$)

The $z^4(z)$ model for nlgYM$_2$ is a state in which the density function in WCP has a minimum at the origin and two maxima which are symmetric with respect to the origin. So one can use the results of the previous section to arrive at

$$\int_c^b \left\{2\hat{A}z^3 - P\int_{-a}^{a}\frac{u_s(t)\mathrm{d}t}{z-t}\right\}\mathrm{d}z = 0, \tag{69}$$

$$\hat{A}(a^2+b^2+c^2) = 2\int_c^b \frac{t\mathrm{d}t}{T(t)}, \tag{70}$$

$$\hat{A}\left\{(a^2b^2 + a^2c^2 + b^2c^2) + \frac{3}{2}(a^4+b^4+c^4)\right\} = 2 + 4\int_c^b \frac{t^3\mathrm{d}t}{T(t)}. \tag{71}$$

Finally, by making use of (54), (59) and (62), it is seen that

$$\begin{aligned} F'_s(A) = V'(A\kappa_s)\Bigg\{&\frac{\hat{A}}{16}\Big[\frac{5}{4}(a^8+b^8+c^8) \\ &- \frac{1}{2}(a^4b^4 + a^4c^4 + b^4c^4) \\ &+ (a^2b^2c^4 + a^2c^2b^4 + a^4b^2c^2) \\ &- (a^2b^6 + a^2c^6 + b^2a^6 + b^2c^6 + c^2a^6 + c^2b^6)\Big] \\ &+ \frac{1}{8}[a^6 + b^6 + c^6 \\ &- (a^2b^4 + a^2c^4 + b^2a^4 + b^2c^4 + c^2a^4 + c^2b^4) \\ &+ 2a^2b^2c^2]\int_c^b \frac{t\mathrm{d}t}{T(t)} \\ &+ \frac{1}{4}[a^4 + b^4 + c^4 \\ &- 2(a^2b^2 + a^2c^2 + b^2c^2)]\int_c^b \frac{t^3\mathrm{d}t}{T(t)} \\ &+ (a^2+b^2+c^2)\int_c^b \frac{t^5\mathrm{d}t}{T(t)} - 2\int_c^b \frac{t^7\mathrm{d}t}{T(t)}\Bigg\}. \end{aligned} \tag{72}$$

By using the same procedure as used in [12,13] and expanding (69)–(72) near the critical point and then solve them together, we obtain

$$F'_s(A) = \frac{V'_s}{\hat{A}}\left[\frac{1}{4} + \frac{\beta}{27}\alpha^2 + \ldots\right], \tag{73}$$

or

$$F'_s(A) - F'_w(A) = \frac{\beta}{27A_c}\alpha^2 + \ldots, \tag{74}$$

where

$$\alpha = \left(\frac{A - A_c}{A_c}\right)^2, \tag{75}$$

and

$$\beta = \left(1 + \frac{A_c\kappa_{cs}V''_{cs}}{V'_{cs}}\right)^2. \tag{76}$$

It is seen that the theory for the ϕ^4 model has a third order phase transition, which is in agreement with ordinary gYM$_2$.

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